On the application of the variational iteration method to a prey and predator model with variable coefficients

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Abstract

We discuss an amazing prey–predator model with variable coefficients, analyze its predictions and the accuracy of the variational iteration method used to solve the nonlinear equations.

1 Introduction

There has recently been great interest in the application of several approximate procedures, like the homotopy perturbation method (HPM), the Adomian decomposition method (ADM), and the variation iteration method (VIM), to a variety of linear and nonlinear problems of interest in theoretical physics [1–15]. In a series of papers I have shown that most of the results produced by those methods are useless, nonsensical, and worthless [16–18]. From now on I will refer to those variation and perturbation approaches as VAPA.

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The purpose of this communication is to analyze a recent application of the VIM to a model for the evolution of a prey–predator system [14] .

2 The model

Yusufoğlu and Erbaş [14] have recently applied the VIM to a prey–predator model with variable coefficients

$$\begin{aligned} \dot{x}(t) &= a(t)x(t) - b(t)x(t)y(t),\ x(0) = \alpha \\ \dot{y}(t) &= -c(t)y(t) + d(t)x(t)y(t),\ y(0) = \beta \end{aligned} \tag{1}$$

According to the authors $x(t)$ and $y(t)$ represent the populations of rabbits and foxes, respectively. Instead of constants a, b, c, d for the growth rate of the prey, the efficiency of the predator's ability to capture the prey, the death rate of the predator, and the growth rate of the predator, the authors introduce functions of time [14] . The authors do not mention any physical, zoological or ecological reason for the substitution of functions for constants, and at first sight it seems arbitrary and unjustified.

In the first example Yusufoğlu and Erbaş [14] choose $a(t) = b(t) = -t$, $c(t) = d(t) = t$, $\alpha = \beta = 2$. In principle, there is no reason for this choice of time–dependent coefficients, except that the equations can be solved exactly. The exact solutions for this model

$$\begin{aligned} x(t) &= \frac{2}{[2 - \exp(t^2/2)]} \\ y(t) &= \frac{2}{[2 - \exp(t^2/2)]} \end{aligned}$$

predict that the population of rabbits always equals the foxes one and both tend to infinity as t approaches $t_c = \sqrt{2\ln 2}$ from the left. The reader may think that this behaviour of the model is completely unrealistic because no population becomes infinity in the world we perceive. However, Yusufoğlu and

Erbaş [14] did not find it unreasonable, and for the time being we may assume that the planet is infinitely large and can accommodate infinite populations of rabbits and foxes. Or that it takes place in another world where such a curious behavior may be possible. It is more difficult to understand that when $t > t_c$ both populations jump to $-\infty$ and then approach zero from below as $t \to \infty$. I am not smart enough to explain this behaviour of the extraterrestrial rabbits and foxes and therefore I have decided to describe it without further analysis.

Yusufoğlu and Erbaş [14] applied the VIM and obtained increasingly accurate approximations $x_j(t)$ and $y_j(t)$ for $j = 1, 2, 3$; for example

$$\begin{aligned} x_3(t) = 256 &+ \left(\frac{1882}{3} - 160t^2 - 64t^4\right) e^{-t^2/2} - \left(72 + 32t^2\right) e^{-3t^2/2} \\ &- \left(804 + 416t^2 + 64t^4\right) e^{-t^2} - \frac{16}{3} e^{-2t^2} \end{aligned} \tag{2}$$

We appreciate that this approximate solution does not exhibit the pole of the exact answer, and tends to zero as $t \to \infty$. Yusufoğlu and Erbaş [14] cleverly overcome this difficulty by comparing their approximate expression and the exact solution only within the interval of reasonable agreement. Besides, notice that the VIM corrects the curious behaviour of the extraterrestrial rabbits and foxes and makes them extinct after a sufficiently long time.

Naively, I tried Padé approximants and found that the simplest ones give much better results than the elaborated combinations of polynomials and exponentials of Yusufoğlu and Erbaş [14]. The fact is that the t–power series expansion of the solution converges for all $t < t_c$, and the Padé approximants constructed from it take into account the poles of the solution as zeros of the denominator [19]. For example, the simple and straightforward [2/4] Padé approximant

$$[2/4](t) = \frac{8(t^2 - 6)}{(t^4 + 16t^2 - 24)} \tag{3}$$

built from the time–series solutions to the model equations yields much more accurate results than the VIM equation (2). It seems to be considerably easier to obtain the power series and their Padé approximants than the application of the VIM. However, you cannot publish the much more reasonable former approach because it is just a textbook example for students. It is worth mentioning that the approximants $[1/2](z)$, $[2/3](z)$, $[3/4](z)$ and $[4/5](z)$, $z = t^2$, exhibit poles at $t^2 = 1.380831519$, $t^2 = 1.386322332$, $t^2 = 1.38629429$, and $t^2 = 1.386294361$, respectively, that clearly converge towards the exact pole $t_c^2 = 1.386294364$. The VIM solutions do not exhibit this property and therefore fail to follow the exact solution as t increases. However, I have been much surprised at the revelation that part of the scientific community does not want to be bothered by such mathematical details.

The second example clearly shows that the unparalleled insight of Yusufoğlu and Erbaş [14] is beyond any mortal's perception of the very nature of the universe. They masterfully choose

$$\begin{aligned} a(t) &= 4 + \tan t, b(t) = \exp(2t), c(t) = -2, d(t) = \cos t, \\ \alpha &= -4, \ \beta = 4 \end{aligned} \tag{4}$$

First of all, notice the initial negative population of rabbits $\alpha = -4$!! The exact solutions

$$\begin{aligned} x(t) &= -4/\cos t, \\ y(t) &= 4e^{-2t} \end{aligned} \tag{5}$$

show that the population of rabbits remains negative and tends to $-\infty$ as $t \rightarrow (\pi/2)^-$, then it jumps to plus infinity and starts decreasing as t increases. On the other hand, the population of foxes tends to zero exponentially, probably due to the stress caused by the negative population of rabbits and their sudden emerging to real world.

Once again, Yusufoğlu and Erbaş [14] apply VIM and obtain third–order ap-

proximate populations. I beg the reader to have a look at the authors' Fig. 2 to appreciate the remarkable performance of the VIM. If that science overload does not exhaust the reader's mind, he/she may then compare the approximate expressions of Yusufoğlu and Erbaş [14] with the naive Padé approximants

$$
\begin{aligned}
x[2/4](t) &= \frac{-16(t^2+30)}{3t^4-56t^2+120} \\
y[3/4](t) &= \frac{-4(4t^3-30t^2+90t-105)}{2t^4+16t^3+60t^2+120t+105}
\end{aligned}
\tag{6}
$$

which are far simpler than the VIM solutions and give much better results. For that reason they are unsuitable for publication.

Yusufoğlu and Erbaş [14] also applied the VIM to models with constant coefficients. Those models are more realistic from a zoological point of view. However, the authors do not show their results and simply mention that they agree with those obtained by the HPM [1]. For example, case I is given by $a = b = 1$, $d = 10c = 1$, $\alpha = 14$, $\beta = 18$ [1,14]. This model exhibits two stationary points: a saddle point at $(x_s, y_s) = (0, 0)$ and a center at $(x_s, y_s) = (1/10, 1)$. The authors do not attempt to reproduce the overall picture of the model dynamics, which is what really matters [19], and restrict themselves to a time interval about the origin because the VIM results will prove entirely useless otherwise.

In Fig. 1 we show the populations given by the expressions derived by Rafei et al [1] by means of the HPM and the exact (numerical) results. We clearly appreciate that both the VIM (Fig. 3 of Yusufoğlu and Erbaş [14]) and the HPM [1] are far from giving a reasonable picture of the behaviour of the prey and predator populations. The same conclusion holds for the other cases treated by Rafei et al [1]. However, if you restrict to the initial time when the animals begin to make acquaintances, then the VAPA results [1, 2, 14] are not too bad (see my earlier discussion of the subject [16]). The fact that nobody in the field of population dynamics is interested in the initial evolution of the system has remained unnoticed in most VAPA applications [1, 2, 14].

3 Conclusions

It is amazing the amount of nonsensical VAPA papers that have recently been published on the treatment of all kinds of linear and nonlinear problems. It is surprising the increasing interest of part of the scientific community in remarkably useless results. The list below shows only those I had time to peruse. The work discussed in this communication is just an example. The reader may also have a look at my previous analysis of other papers**??**.

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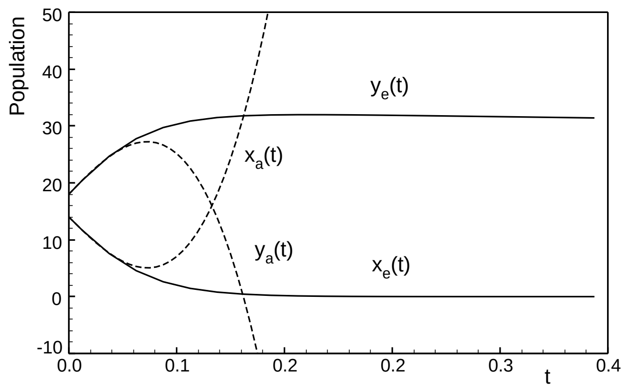

Fig. 1. Exact (e) and approximate (a) populations for the model with constant coefficients